Numerical Spin Analysis of Relativistic Bondi Accretion in M87*

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Abstract

We use a general relativistic numerical hydrodynamics code, Athena++, to simulate a Bondi Accretion model of M87 with a varying spin parameter. By varying the parameters of black hole M87* with spin 0.0, 0.4, and 0.95, we assessed how increasing spin affects 5 critical variables: radial velocity, density, pressure, total energy, and radial momentum. We found that spinning black holes disrupt spherical symmetry, but reach a steady state solution over time. Additionally, we find that supersonic gas flow is more turbulent with higher spin values, which generates shocks inside the sonic point radius.

Introduction

Bondi Accretion models spherically symmetric accretion upon a compact object. Mass Accretion in this model occurs at a rate:

$$\dot{M} \simeq \pi R^2 \rho v$$

Where ρ is the ambient density, v is the radial velocity of the fluid, and R refers to the radial coordinate[9]. We define the critical Bondi radius r_sonic by setting the escape velocity equal to the sound speed:

$$r_s = \frac{GM}{2c_s^2}$$

This equation represents the boundary between subsonic and supersonic infall. We can use this equation to estimate the gas accretion onto black holes, particularly the supermassive black hole M87* (center of the elliptical galaxy M87). M87* is estimated to have a spin parameter of around 0.4, but we explore how its behavior would change if the spin were much less or much more than this value. Black holes are defined by their mass, spin, and charge (we treat M87* as uncharged), so by holding mass constant at M87*'s estimated value of 2.4 billion solar masses[5], we can test the direct effects of spin on the accreting gas.

Modelling the accretion of gas onto a compact object allows us to make predictions about what we should expect to observe in the center of the galaxy M87.

Methods

Athena++ is a free, open source C++ program capable of simulating compressible and relativistic magnetohydrodynamics. It uses Adaptive Grid Mesh Code to solve partial differential equations in each grid element for each increment in time[1]. The C++ code solves the Navier-Stokes fluid equations: mass continuity, momentum conservation, and energy conservation.

$$\begin{aligned}
&\textbf{Mass:} && \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &&= 0 \\
&\textbf{Momentum:} && \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) &&= -\nabla p + \nabla \cdot \tau + \mathbf{F} \\
&\textbf{Energy:} && \frac{\partial}{\partial t}\left[\rho\left(e + \frac{1}{2}u^2\right)\right] + \nabla \cdot \left[\rho \mathbf{u}\left(e + \frac{1}{2}u^2\right)\right] &&= \nabla \cdot (k \nabla T) + \nabla \cdot (-p\mathbf{u} + \tau \cdot \mathbf{u}) + \mathbf{u} \cdot \mathbf{F} + \mathcal{Q}
\end{aligned}$$

[12]

To implement the code, we configure a C++ problem generator, modify the input file parameters, run the simulation with python, and output the data as hdf5 files, We then assemble plots using the derived fields[11].

We set one code mass of M=1, the spin parameter of a=0.0, 0.4, 0.95, and used a tstep=0.5 and tlim=60. Then we converted the code units using M87* parameters, where 1 M corresponds to 2.4e9 solar masses. From here, we use the equation for the sonic point radius to approximate r_s = 0.15kpc (8 code units of R (radius)) and one code unit corresponding to 0.002 Myr. Using yt and matplotlib, we plotted Athena's derived variables using the conversion factors listed above.

We simulate general relativistic Bondi Accretion using the kerr-schild metric, which adjusts for axially symmetric geometry for a spinning, non-charged black hole. Our problem is configured with Kerr-Schild coordinates.

The Kerr-Schild metric defined such that:

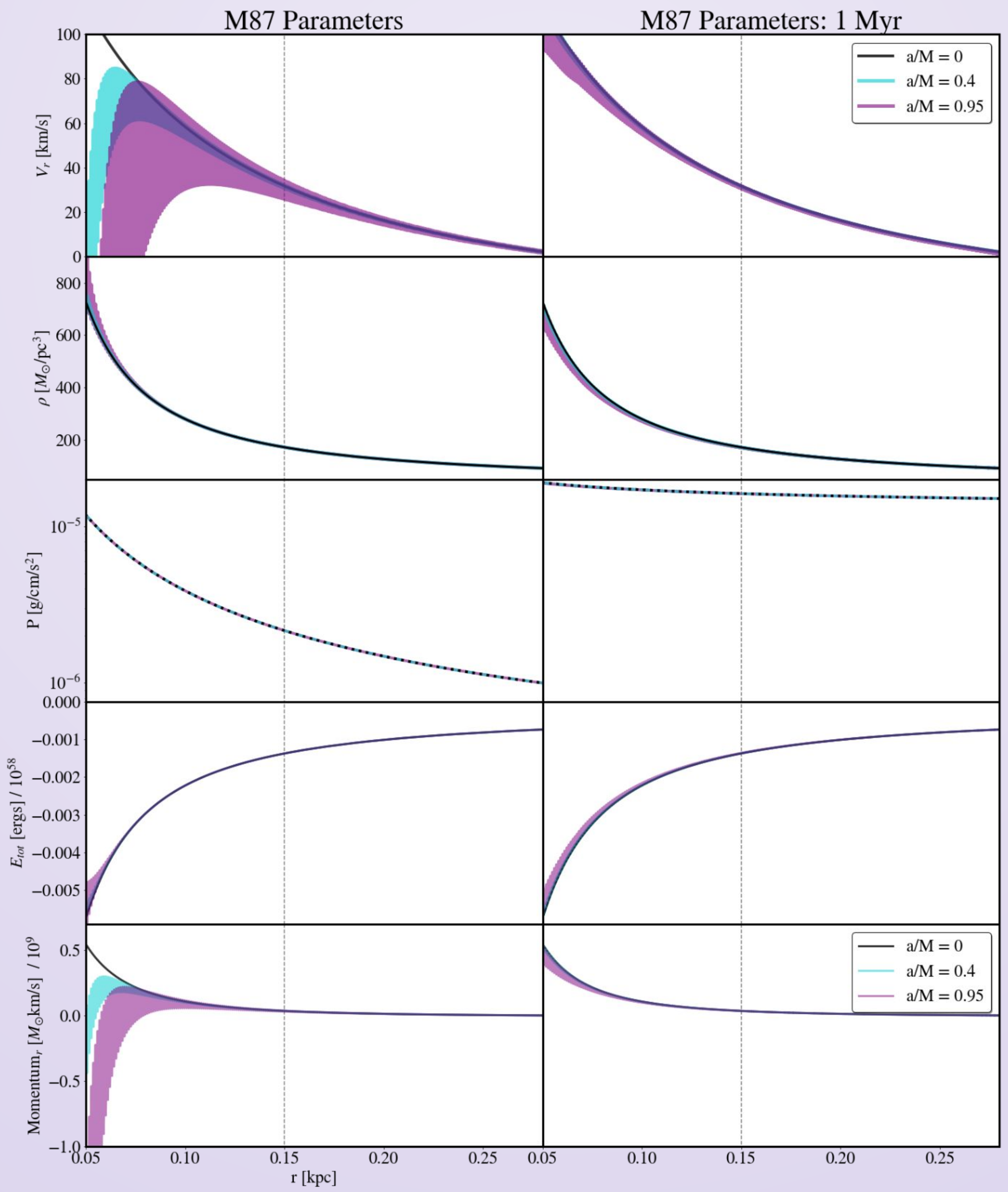
$$g_{\mu\nu} = \eta_{\mu\nu} + f k_\mu k_\nu \qquad f = \frac{2GMr^3}{r^4 + a^2 z^2}$$

$$\mathbf{k} = (k_x, k_y, k_z) = \left(\frac{rx + ay}{r^2 + a^2}, \frac{ry - ax}{r^2 + a^2}, \frac{z}{r}\right) \qquad k_0 = 1$$

Analysis & Results

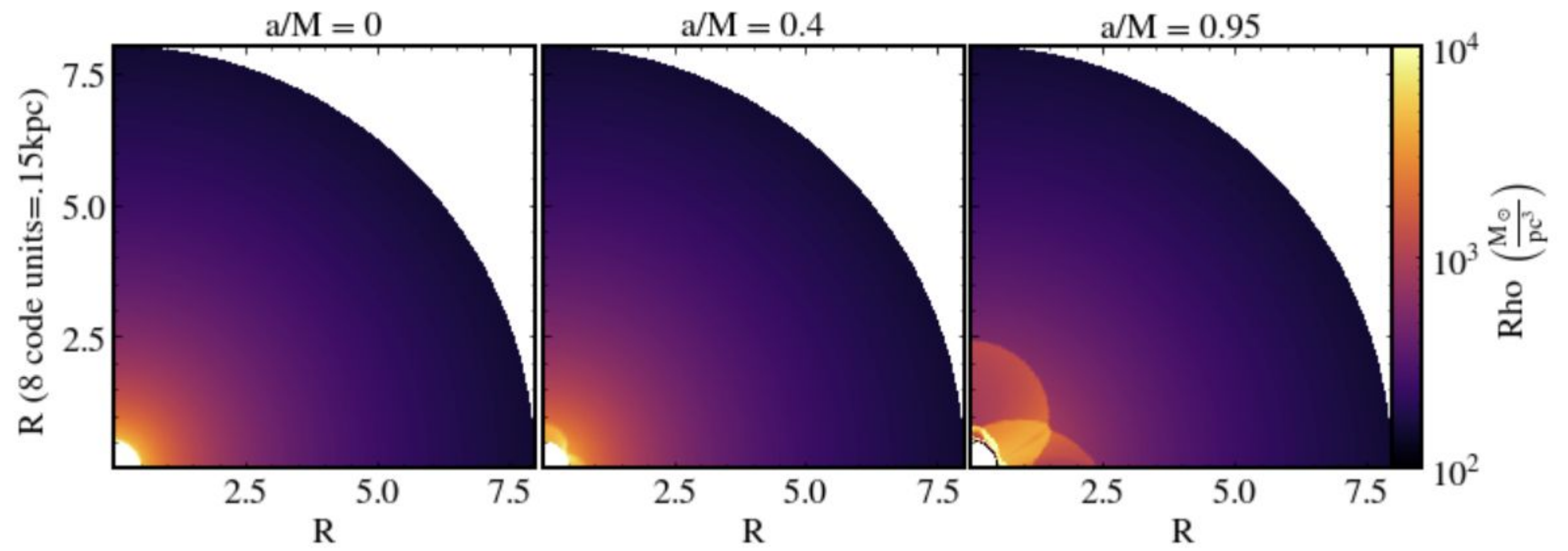
Although a nonzero spin factor initially breaks spherical symmetry, the Bondi Accretion model inevitably leads to a steady state solution. We have created variable profiles for the initial and final conditions of 5 crucial quantities (primitive and conserved). Each system reaches a steady state by 1 Myr, so we choose this to be our final frame.

The velocity profile at t=0 shows a high spread of velocity between the changing-spin systems close to the black hole. This indicates that centripetal force changes the direction of the velocity towards the tangential direction. However, when each of the systems has achieved stability at 1 Myr, the velocities are once again gravitationally- and thus radially-dominated.



For the above figure, radial velocity, density, pressure, total energy, and radial momentum profiles for the initial conditions of an M87*-like black hole are shown over the radial distance from the center of the black hole. The critical radius lies at 0.15 kpc from the black hole, signified by a vertical dashed line. The left column shows the initial conditions (t=0) while the right column shows the profiles after 1 Myr has passed(t=60 code units~1 Myr).

Scan this QR code to see M87* accretion animated!



The figure above compares the turbulence of each black hole at 0.29 Myr . The zero spin system has reached its steady state, the 0.4 spin system is just reaching steady state, and the 0.95 system is still turbulent. Black holes with higher spin require a longer timescale to achieve their steady state than those with lower spin. A black hole with zero spin (Kerr black hole) achieves steady state gas flow immediately. We define code units of length such that 8 units = 0.15 kpc.

For M87*, the sonic point radius (where the Mach number exceeds 1) is estimated to be at r=0.15kpc. Inside the sonic point radius, turbulent flow of the supersonic gas incites shock wave collisions. We can see shock waves in the spinning black hole models where the gas lobes collide, but we see no shock waves in the static black hole because of its immediately laminar flow.

Conclusion

We find that the Bondi Accretion model for spinning black holes induces a steady state solution in each case of spin (a/M=0.0, 0.4, 0.95). Black holes with higher spin take longer to reach this steady state, and have more turbulence, which causes a higher production of shock waves within the critical radius. As seen in the evolution of the velocity profile, we also find that increasing spin of an M87-like black hole weakens the radial component of velocity initially, but restores in magnitude as the system reaches a steady state solution. Over time, the gravitational pull of the black hole nullifies the centripetal force caused by spin.

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