



# Determining $H_0$ from Time Delays in Lensed Quasars

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## Background

### [Gravitational Lensing]

- Einstein's theory of general relativity: gravity is not a force, but the result of space and time distorting around a massive body
- Light rays that pass near a massive body (e.g. black hole or galaxy) are deflected. A light source, aligned with a gravitational lens produces an illusion to an observer that the source is in a different location.
- Three types of gravitational lenses: microlenses, weak lenses, and strong lenses.
- Strong lenses produce multiple images which is important for time delay calculations.

### [Hubble's Law]

- Hubble's Law (expanding universe): an object's recessional velocity is proportional to its distance from Earth.
- The Hubble parameter  $H(t)$  acts as the constant of proportionality. We can trace this parameter back to the Big Bang,  $H(0)$ , giving us insight into the age of the universe.
- Calculating  $H_0$  is an important open problem in cosmology.
- Multiple techniques produce conflicting results. Gravitational lensing provides an independent way of measuring  $H_0$ !

### [Time Delays]

- Light curve of a body: the intensity of light we receive from it over a period of time.
- The positioning of the source and lens affects the distance that light from two or more images travels to reach the observer and hence the time delay between them [8].
- Additionally, the time delay depends on the mass distribution of the lens. It takes light longer to travel through more massive regions because the space-time is more bent.

## Theory

We calculate Hubble's Constant for lensing systems with galactic lenses and quasar sources. Refsdal derives the following equation for the Hubble Constant [7-8]:

$$H_0 = \frac{Z_S Z_B \alpha (\alpha_1 - \alpha_2)}{\Delta t (Z_S - Z_B)}$$

Here,  $Z_S$  and  $Z_B$  represent the redshifts of the source and lens, respectively. The angle between the lens and the first image, the source and the second image, and their sum are given by  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha$ , while  $\Delta t$  is the time delay. This equation was derived for the case that the lens is a star, therefore, it is only valid under the following assumptions:

- The redshifts of the lens and source are small such that Hubble's Law applies,
- The deflecting galaxy is spherically symmetric,
- The motion of the system is radial with respect to the observer.

Because the observed redshifts are not small for gravitationally lensed quasars, a relativistic correction was made using the following equation:

$$H_0 = \frac{Z_S \alpha (\alpha_1 - \alpha_2)}{\Delta t (Z_S - Z_B)} \cdot \frac{Z_B (Z_B + 2)}{Z_B (Z_B + 2) + 2}$$

Using right ascension ( $R_1$ ) and declination ( $D_1$ ) measurements for the lens and images,  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha$  are calculated using the following equation derived to compute the angular separation ( $A$ ) between two celestial objects [3]:

$$\cos(A) = \sin(D_1) \cdot \sin(D_2) + \cos(D_1) \cdot \cos(D_2) \cdot \cos(R_1 - R_2)$$

## Methodology

We selected the lensing systems SDSS J1001+5027, SDSS J1339 +1310, and WFI2033 -4723 based on availability of redshift, position, and time delay data [2]. Because of the extensive observation time required, only a handful of lensed quasar time delays have been documented [1]. Additionally, many redshifts were reported with great uncertainty. These were the limiting factors in our selection process.

PyCS3 is the main software used in order to estimate time delays from given light curves [5, 10-11].

We imported the light curve data for systems SDSS J1001+5027 and SDSS J1206+4332 from the COSMOGRAIL website.

- To represent the light curves as functions of data, we microlensed them using splines.
  - Microlensing occurs when light behind an object is bent due to its gravitational field, causing light from the background object to create distorted, multiple, and brightened images.
  - A spline transforms data to create a nonlinear predictor from a single predictor
- We optimized our splines after achieving our microlensing algorithms.
  - These help to fill in any gaps in the data based off our algorithms
- Finally, we create a histogram of our values which we achieved from bootstrapping methods as well as optimizing splines.
  - This step created hundreds of mock light curves in order to train the algorithms
  - We were able to achieve a range of time delay values.

Light curve data for the other two systems are not available in the COSMOGRAIL database, but we computed a time delay of 113.12 days for SDSSJ1001+5027 which is slightly less than the actual value of 119 days [2].

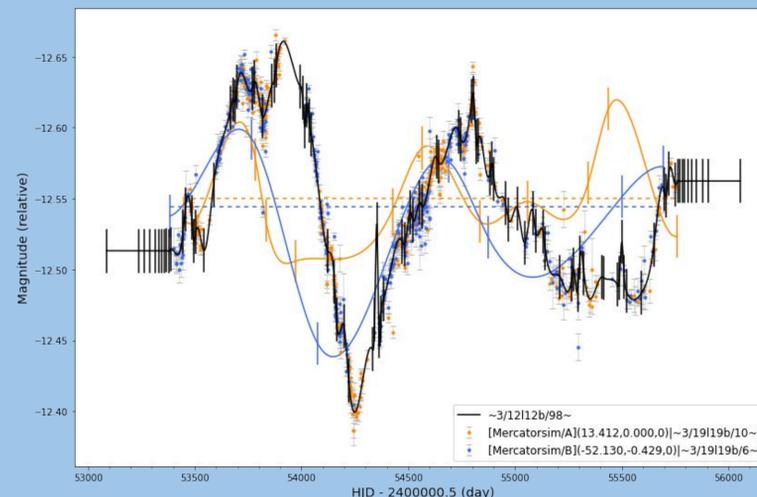


Figure 1: displaying light curves of different images for SDSSJ1001+5227 [5, 10-11]

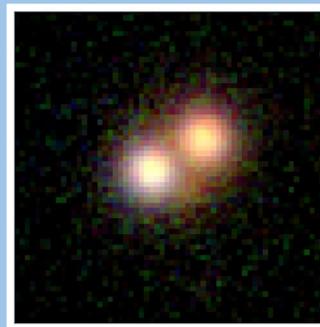


Figure 2 (left): Pan-STARRS image for SDSSJ1001+5027 [2]

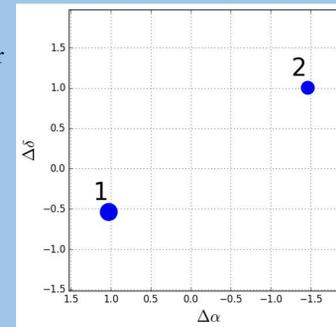


Figure 3 (right): Image positions for SDSSJ1001+5027 [2]

## Results

| System Name     | Type   | $H_0$ (km/s/Mpc) (With relativistic corrections) |
|-----------------|--------|--|
| SDSS J1001+5027 | Double | $58 \pm 13$                                      |
| SDSS J1339+1310 | Double | $68 \pm 18$                                      |
| WFI2033-4723    | Quad   | $80 \pm 25, 62 \pm 19$                           |

## Analysis and Discussion

The small discrepancy in our time delay from the value reported in [2] is due to microlensing representation and optimization processes, which uses random sampling and an insufficient amount of data from which the best fit line is estimated. This contributes a small but non-zero error to our final value for the first system. We computed a wide range of values for  $H_0$  from many lensed quasar systems, the most reasonable results (i.e. close to or within the accepted range of 66-82 km/s/Mpc [12]) are reported in the table above. The significance of these results lies in the fact that the underlying model makes no assumptions about other cosmological parameters and thus these measurements are independent of them. We suspect that the errors for all systems originate from the violation of the assumptions discussed (see Theory); a more extensive evaluation and system selection process would be the next step to decrease uncertainties. Extensive modeling of the lens mass distribution and gravitational potential is required to determine if it is spherically symmetric and thus a candidate for Refsdal's mathematical framework [4]. For example, system SDSS J1001+5027 contains multiple lensing galaxies, suggesting a more complex gravitational potential [9]. In the same light, massive objects near the lens are likely to affect the deflection of light from the quasar source. Finally, light scattering from cosmic dust is likely to influence the observed source luminosities, and ultimately the time delays of the system.

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\*This online database gave access to various papers that computed time delays as well as access links to GAIA.

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