Measuring Cosmic Distances using Gravitational Waves

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Abstract

We present an investigation of the viability of using gravitational wave events as a form of cosmic distance measurement through comparison to established methods, specifically Type Ia Supernova [3].This was done through the comparison of H_0 values calculated with gravitational wave data from LIGO with H_0 values calculated from Type Ia supernova using data from Open Supernova Catalogue [6][7]. We find that the confidence intervals of the H_0 values derived from gravitational wave events and Type Ia supernova overlap, suggesting that gravitational waves may be an effective means of cosmic distance measurement. This overlap also demonstrates the viability of the method of gravitation wave inclination angle calculation using a fitting function that uses the luminosity distance provided by LIGO.

History of calculating H_0

H_0 can be described as the rate at which the recessional velocity of an object increases over distance. This requires knowledge of both an object's recessional velocity and it's distance. Velocities can be derived through observation of redshift and traditional methods of Hubble constant calculation have utilized the cosmic distance ladder to derive distances. Type Ia supernova observation has given us accurate distance measurements due to them being standardizable candles. However, errors may be introduced from extinction due to intergalactic dust and other external errors.

$$v = H_0 d_L$$

Gravitational waves (GW) offer astronomers a new way of observing the universe, and detailed analysis of gravitational waves could reveal information about the distance to the source of the GW. Unlike light, gravitational waves are not affected by interstellar dust clouds, making them a prime target to use to explore the Hubble parameter. New methods to determine the Hubble parameter are especially important during this time in cosmology because the Hubble constant calculated from the cosmic microwave background is no longer consistent with that calculated from the cosmic distance ladder. New independent methods of calculating H_0 may then shed some light on the source of this tension.

Supernovae Theory

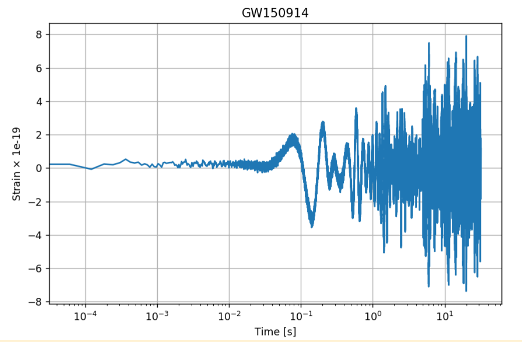
Type Ia supernovae (SNe Ia) are stellar explosions with standardizable peak luminosity. When observing SNe Ia spectra, we see fluctuations in the SNe's luminosity at each wavelength. The absorption lines, parts of the curve that significantly dip, correspond to the wavelengths of various elements. The SNe's light will be stretched due to the SNe's recessional velocity, causing these observed wavelengths to be longer the rest wavelengths--assuming a positive recessional velocity. Therefore, the recessional velocity can be derived by comparing the elements' observed wavelength in the SNe spectra to their corresponding rest wavelengths. The relationship between the redshift and the recessional velocity can be summarized as v= cz for relatively low redshifts [2]. We can then use the SNe's magnitude to calculate it's distance from the distance-modulus equation, which compares the absolute magnitude to the apparent magnitude to give a distance. We can then derive H0 by dividing the SNe's recessional velocity by its distance.

Gravitational Wave Theory

$$d_{new} = n d_{old} = n \left[\frac{2c}{h_+} \left(\frac{G\mathcal{M}}{c^3} \right)^{5/3} \Omega^{\frac{2}{3}}(t) \left(1 + cos^2 i\right) cos 2\Phi(t) \right]$$

$$cos^2 i = 2nh_+ - 1 \rightarrow \boxed{i \approx cos^{-1}\left(\sqrt{|2nh_+ - 1|}\right) + 2\pi k}$$

GW's can be thought of as a longitudinal ripple in space that travels at the speed of light. These ripples occur when two massive objects collide and coalesce into a singular, more massive object, causing the distance between two points in space to change periodically according to the frequency of the signal. The amount of separation between two points is referred to as the strain of the gravitational wave, which is also the amplitude of the GW. In order to calculate distance, we need the chirp mass, strain, and inclination angle, while also accounting for the build of instrumental errors when calculating the frequency of the GW. Due to time constraints, we used maximum values throughout our work, such as the combined mass rather than the chirp mass, and maximum strain in order to minimize the amount of noise in the data and determine a rough estimate for the distance to each GW source and its respective H_0 value.

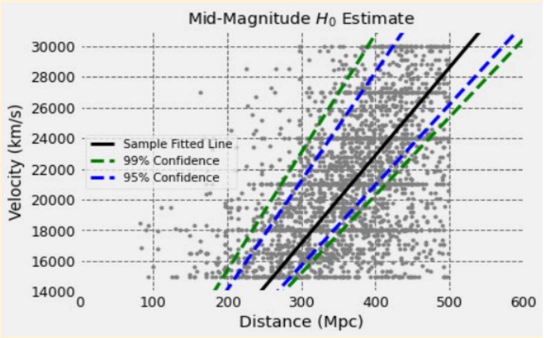


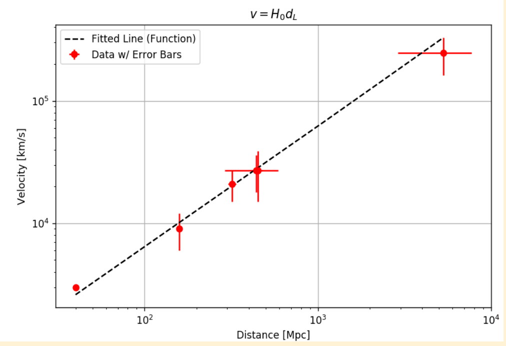
Methods

We use a regression function to find a best fit line relating the recessional velocity and distance of 2677 Type Ia supernovae that were within the same redshift values as the gravitational wave sample. The slope of the best fit line was then taken as the Hubble constant. The redshift values and apparent magnitude were provided by the Open Supernova Catalog [6] and the distances were derived using the upper, middle, and lower bounds of the average absolute magnitude of Type Ia supernova, giving us three Hubble constants [8]. (We did not use the more accurate method of template fitting to derive absolute magnitude due to time and technical constraints.) Confidence intervals for each Hubble constant were then constructed using a bootstrap of 1000.

We found an expression to determine the distance to GW events, which relates distance to the strain, combined mass, inclination angle, and the buildup of errors as the wave propagates [3]. We created a model based on this expression and found out that it only worked for 2/10 GW signals. To improve our model, we defined the dimensionless constant n where n = d_{old} / d_{LIGO} , which is a measure of how far off our distance measurements are from LIGO's luminosity distance measurements. We derived an inclination angle equation which allowed us to use n as an amplifier on the strain [4]. This allowed us to get gravitational wave distance and inclination angle results for 6/10 signals. We used the redshift values reported by LIGO and our calculated distance values to determine the expansion rate of the universe.

Results





SNe Results:

Absolute Magnitude	99% Confidence Lower	95% Confidence Lower	H_0 $(kms^{-1} Mpc^{-1})$	95% Confidence Upper	99% Confidence Upper
Min Mag (-19.26+.16)	53	55.5	60.49	74.36	85.04
Mid Mag (-19.26)	50.71	52.36	57.18	70.74	76.97
Max Mag (-19.26-.16)	48.89	49.87	54.16	63.24	69.8

GW Hubble parameter:

Circumstances	H0 Values (km/s/Mpc)
str16	str16
With Kilonova	75.056 ± 0.2048
Without Kilonova	58.744 ± 10.2409

GW Distance Calculator Results:

Signals	LIGO Distances (Mpc)	Calculated Distances (Mpc)	Calculated Inclination Angles (rad)	H0 Values (km/s/Mpc)
str19	int64	str17	float64	str15
GW170817 (kilonova)	40	39.966 ± 7.0	2.652	75.063 ± 0.205
GW190425	159	159.001 ± 69.0	3.054	56.603 ± 18.87
GW170608	320	320.145 ± 120.0	0.528	65.595 ± 19.082
GW150914	440	439.46 ± 150.0	1.225	61.439 ± 29.31
GW151226	450	450.039 ± 180.0	0.616	59.995 ± 26.94
GW190521	5300	5294.293 ± 2400.0	0.892	46.465 ± 26.367

Discussion and Future Work

The GW Distance calculator successfully determined the distance, inclination angle, and Hubble parameter for 6/10 of the gravitational waves detected by LIGO. With the error bounds of the gravitational wave H_0 estimates overlapping with the SNe results, they are shown to be a viable form of calculating H_0. However, the current uncertainties on the GW H_0 results are too large to provide any insight into the source of the cosmological crisis, where the CMB estimates and the cosmic distance ladder estimate diverge. Determining the distance and inclination angle of the four signals where our calculator failed would require higher resolution strain data. Higher quality data would also allow us to explore the cause of the inclination angle degeneracy problem in greater detail.

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